Performance Model for Distributed Sonar Tracking

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ABSTRACT

The objective of this paper is to motivate the two-stage approach to target tracking for a field of sensors with variable detection performance, and to provide a modeling tool to predict tracking performance. In the two-stage approach, each sensor performs its own target tracking, followed by an automated track fusion capability. We motivate the two-stage approach by developing a Markov chain model for the multi-stage tracking process, and analyzing its performance. Earlier Markov chain modeling work applied to target tracking can be found in [1-2]. In this analysis, all sensors share a common coverage area. Thus, this paper does not address the benefit of multiple sensors in terms of area coverage, nor does it address improved localization accuracy.

1 INTRODUCTION

This paper motivates the two-stage approach to target tracking for a field of sensors with variable detection performance, and provides a modeling tool to predict tracking performance. In the two-stage approach, each sensor performs its own target tracking, followed by an automated track fusion capability. This paper is organized as follows. In Section 2, we introduce the target and sensor models used in this analysis. In Section 3, we introduce the single-stage tracker model. Section 4 discusses the two-stage tracker, which includes a concatenation of models for the first stage (the single-sensor tracker model), and the second stage (the track fusion model). Section 5 provides a simulation-based performance evaluation of the tracking approaches. Conclusions and recommendations for further research are in Section 6. In particular, it is of interest to compare our model-based conclusions with results based on actual single-stage and two-stage tracking algorithms.

2 TARGET AND SENSOR MODELS

We model target motion with a standard nearly constant velocity model. We model target detections to be noisy positional measurements, with a constant measurement covariance matrix R. We assume that for each target-sensor pair, detection statistics are governed by a two-state Markov chain, with high-detection and low-detection states. This is a simple model for variable detection performance due to the source-target-receiver geometry and environmental effects. The model is illustrated in Figure 1. We denote the detection state by X_D. The detection probability is a function of the detection state X_D.

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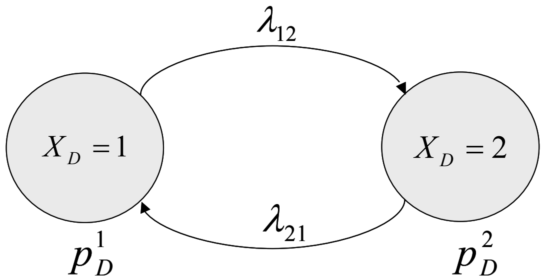

Figure 1: Markov chain model for detection state.

Note that the detection model is defined in continuous time. The probabilities of transition from one state to the other from the k^{th} to the $(k+1)^{st}$ scan are given approximately by equations (2.1-2.2) below, where $\Delta t_k = t_{k+1} - t_k$ is the time between scans.

$$p_{12}(\Delta t_k) = 1 - \exp(-\lambda_{12}\Delta t_k), \tag{2.1}$$

$$p_{21}(\Delta t_k) = 1 - \exp(-\lambda_{21}\Delta t_k). \tag{2.2}$$

In addition to possible target detection, in each data scan we have uniformly distributed false alarms in measurement space, with density λ.

3 SINGLE-STAGE TRACKER MODEL

In the single-stage tracking approach, each sensor (i.e. each source-receiver pair) generates a time-ordered sequence of scans, and the sequences are merged into a single time-ordered sequence of scans. Then, the tracking algorithm processes the scans sequentially. Since for the same target the detection state varies across sensors, it is simplest to model the detection probability to be constant and equal to its weighed average based on the stationary probability distribution for the detection state X_D. This stationary distribution is given by the following:

$$\pi_D^1 = \Pr(X_D = 1) = \frac{\lambda_{21}}{\lambda_{21} + \lambda_{12}}, \tag{3.1}$$

$$\pi_D^2 = \Pr(X_D = 2) = \frac{\lambda_{12}}{\lambda_{21} + \lambda_{12}}. \tag{3.2}$$

Then, we have:

$$P_D = \pi_D^1 P_D^1 + \pi_D^2 P_D^2. \tag{3.3}$$

Our track initiation is based on *M-of-N* confirmation logic. The following finite state Markov chain describes the dynamics for the track logical state X_L, for 3-of-4 confirmation logic.

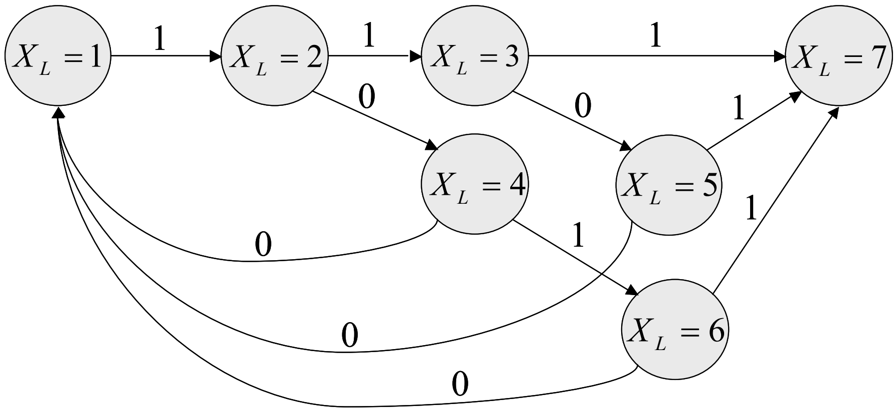

Figure 2: Markov chain model for track logical state, with 3-of-4 confirmation logic.

Before any data is processed, each (future) tentative track is conceptually in logical state $X_L = 1$. Subsequently, detection events (indicated with a 1) and missed detection events (indicated with a 0) lead to logical state transitions. If the *M-of-N* criterion is achieved, the track is confirmed. In the example in Figure 2, this corresponds to logical state $X_L = 7$.

The data association gating (or validation) parameter leads to a probability P_G that a detection can be feasibly associated to a track. Thus, detection events occur with probability $P_D P_G$, and non-detection events occur with probability $1 - P_D P_G$. Thus, for the *target present* case, the probability transition matrix A_D for the logical state Markov chain model is given by the following (in the case of *3-of-4* confirmation logic):

$$A_D = \begin{bmatrix} 1-P_D & P_D & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & P_D P_G & 1-P_D P_G & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-P_D P_G & 0 & P_D P_G \\ 1-P_D P_G & 0 & 0 & 0 & 0 & P_D P_G & 0 \\ 1-P_D P_G & 0 & 0 & 0 & 0 & 0 & P_D P_G \\ 1-P_D P_G & 0 & 0 & 0 & 0 & 0 & P_D P_G \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{3.4}$$

We define $\pi_L(k) = \left[\pi_L^1(k) \quad \ldots \quad \pi_L^n(k)\right]$ to be the expected probability distribution on logical track states after the kth scan; n is the number of track logical states. All tracks start in state $X_L = 1$, so that $\pi_L^1(0) = 1$. The probability distribution evolves as follows:

$$\pi_L(k+1) = \pi_L(k) A_D . \tag{3.5}$$

The average track confirmation time is given by:

$$T_D = \lim_{K\to\infty} \frac{\sum_{k=1}^{K}\left[\left(\sum_{j=1}^{k}\Delta t_{j-1}\right)\cdot\left(\pi_L^n(k)-\pi_L^n(k-1)\right)\right]}{\pi_L^n(K)}. \tag{3.6}$$

In addition to the average track confirmation time in the case of target presence, we are interested to determine the average number of new false tracks per unit time (hours), which we denote as N_{FT}. This can be determined approximately as the product of Λ, the average number of false alarms per scan ($\Lambda = \lambda\overline{S}$, where $\overline{S}$ is the size of the surveillance region), the probability that a false alarm leads to a confirmed false track (P_{FT}), and the scan rate *r* (the average number of scans per hour). (Note that we neglect the fact that, due to data association, it may be that not every false alarm is treated as a potential start of a new track). Below, we discuss the determination of P_{FT}.

The size of the validation region will depend on the detection sequence. In particular, the track state covariance $P(k\mid k)$ is defined recursively as follows:

$$P(1\mid 1) = \begin{bmatrix} \multicolumn{2}{c}{\multirow{2}{*}{R}} & 0 & 0 \\ & & 0 & 0 \\ 0 & 0 & \sigma_{\dot{x}}^2 & 0 \\ 0 & 0 & 0 & \sigma_{\dot{y}}^2 \end{bmatrix}, \tag{3.7}$$

$$P(k+1\mid k) = \Phi(\Delta t_k)P(k\mid k)\Phi'(\Delta t_k) + Q(\Delta t_k), \tag{3.8}$$

$$P(k+1\mid k+1) = (I - \delta_{k+1}L(k+1)H)P(k+1\mid k), \tag{3.9}$$

where $\sigma_{\dot{x}}^2$ and $\sigma_{\dot{x}}^2$ reflect *a priori* uncertainty on target velocity, $\delta_{k+1} = 1$ denotes a detection event, $\delta_{k+1} = 0$ is a no-detection event, and

$$R = \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix}, \tag{3.10}$$

$$\Phi(\Delta t_k) = \begin{bmatrix} 1 & 0 & \Delta t_k & 0 \\ 0 & 1 & 0 & \Delta t_k \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{3.11}$$

$$Q(\Delta t_k) = \begin{bmatrix} \frac{1}{3}q_x\Delta t_k^3 & 0 & \frac{1}{2}q_x\Delta t_k^2 & 0 \\ 0 & \frac{1}{3}q_y\Delta t_k^3 & 0 & \frac{1}{2}q_y\Delta t_k^2 \\ \frac{1}{2}q_x\Delta t_k^2 & 0 & q_x\Delta t & 0 \\ 0 & \frac{1}{2}q_y\Delta t_k^2 & 0 & q_y\Delta t \end{bmatrix}, \tag{3.12}$$

$$S(k+1) = HP(k+1 \mid k)H' + R\,, \tag{3.13}$$

$$L(k+1) = P(k+1 \mid k)H'S^{-1}(k+1)\,, \tag{3.14}$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \tag{3.15}$$

Note that, for simplicity, we use a platform-independent positional measurement model. The constants q_x and q_y are process noise parameters. Using the general expression in [1, p. 96], the validation region at scan k+1 is given by:

$$V\left(\delta^k\right) = \gamma\pi \left|S(k+1)\right|^{1/2}. \tag{3.16}$$

The choice $\gamma = 0.92$ leads to $P_G = 0.99$. In (3.16), we make explicit the dependence of the validation region on the detection sequence $\delta^k = (\delta_1, ..., \delta_k)$. The number of false alarms in the validation region at scan k+1 is Poisson distributed with parameter $\lambda V\left(\delta^k\right)$. Thus, the probability of a false alarm in the validation region is given by:

$$P_{FA}\left(\delta^k\right) = 1 - \exp\left(-\lambda V\left(\delta^k\right)\right). \tag{3.17}$$

For each false alarm, we have $\pi_L^2(1) = 1$, i.e. we consider the false alarm to have occurred in the first scan. Then, for the *3-of-4* track confirmation logic illustrated in Figure 2, we have the following probability transition matrix for the track logical state:

$$A_{FA} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & P_{FA}(1) & 1 - P_{FA}(1) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 - P_{FA}(1,1) & 0 & P_{FA}(1,1) \\ 1 - P_{FA}(1,0) & 0 & 0 & 0 & 0 & P_{FA}(1,0) & 0 \\ 1 - P_{FA}(1,1,0) & 0 & 0 & 0 & 0 & 0 & P_{FA}(1,1,0) \\ 1 - P_{FA}(1,0,1) & 0 & 0 & 0 & 0 & 0 & P_{FA}(1,0,1) \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{3.18}$$

Then, we have:

$$P_{FT} = \pi_L^n(N)\,, \tag{3.19}$$

from which we obtain:

$$N_{FT} = \lambda \overline{S} \pi_L^n(N) r\,. \tag{3.20}$$

Note that our tracker model does not use a sliding-window *M-of-N*, in that the first detection defines the time extend of the window and if track confirmation is not achieved, the entire track segment is discarded. A more effective approach (but more difficult to model) is used in our MHT tracker [3].

4 MULTI-STAGE TRACKER MODEL

4.1 Single-Sensor Tracker Model

In the simplified single-stage tracker model described above, we use an average P_D that is based on the stationary probability distribution given by equations (2.3-2.4). For a single-sensor tracker, we estimate the detection probability as a function of the detection sequence. We use standard recursive calculations for finite state Markov chains [4]:

$$\pi_0\left(\delta^0\right)=\left[\pi_D^1 \quad \pi_D^2\right], \tag{4.1}$$

$$\begin{aligned}\pi_{k+1}\left(\delta^{k+1}\right)=\frac{1}{c}\pi_k&\begin{bmatrix}1-P_{12}(\Delta t_k) & P_{12}(\Delta t_k)\\ P_{21}(\Delta t_k) & 1-P_{21}(\Delta t_k)\end{bmatrix}\\ &\cdot\begin{bmatrix}P_D^1+(1-\delta_{k+1})(1-2P_D^1) & 0\\ 0 & P_D^2+(1-\delta_{k+1})(1-2P_D^1)\end{bmatrix},\end{aligned} \tag{4.2}$$

where c is a normalizing constant. Then, the estimated detection probability is given by:

$$P_D\left(\delta^{k+1}\right)=\pi_{k+1}\left(\delta^{k+1}\right)\cdot\begin{bmatrix}P_D^1\\ P_D^2\end{bmatrix}. \tag{4.3}$$

We use equation (4.3) in equation (3.4), rather than a constant detection probability. This results in the following probability transition matrix:

$$A_D=\begin{bmatrix}1-P_D & P_D & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & P_D(1)P_G & 1-P_D(1)P_G & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1-P_D(1,1)P_G & 0 & P_D(1,1)P_G\\ 1-P_D(1,0)P_G & 0 & 0 & 0 & 0 & P_D(1,0)P_G & 0\\ 1-P_D(1,1,0)P_G & 0 & 0 & 0 & 0 & 0 & P_D(1,1,0)P_G\\ 1-P_D(1,0,1)P_G & 0 & 0 & 0 & 0 & 0 & P_D(1,0,1)P_G\\ 0 & 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}. \tag{4.4}$$

Note that transitions from state $X_L=1$ are based on the stationary detection probability, since $P_D\left(\delta^0\right)=P_D$. Equations (3.5-3.6) are unchanged. Also, for single-sensor tracking, there is no change to the calculations for the number of false tracks rate N_{FT} given in Section 3. (Note that generally the scan rate r will be slower in single-sensor tracking, relative to the single-stage multi-sensor approach).

4.2 Multi-Stage Tracking

As a simple model for the multi-stage tracker, we assume that all tracks generated by a number of single-sensor trackers are combined as follows: true tracks are fused, and false tracks are not. Thus, if any system has confirmed target detection, the overall system has confirmed detection as well. The probability of system confirmation at scan k and the average track confirmation time are given below. S is the number of single-sensor trackers. We have added a superscript to identify the i^{th} tracker's probability distribution over logical states π_L^i, and the probability associated with the confirmation state is denoted $\pi_L^{i,n(i)}$.

Indeed, these quantities may vary across trackers as individual tracker parameters and input data characteristics vary.

$$\Pi(k) = 1 - \prod_{i=1}^{S}\left(1 - \pi_L^{i,n(i)}(k)\right), \tag{4.5}$$

$$T_D = \lim_{K \to \infty} \frac{\sum_{k=1}^{K}\left[\left(\sum_{j=1}^{k} \Delta t_{j-1}\right) \cdot \left(\Pi_L^n(k) - \Pi_L^n(k-1)\right)\right]}{\Pi_L^n(K)}. \tag{4.6}$$

The false track rate is given by:

$$N_{FT} = \sum_i N_{FT}^i , \tag{4.7}$$

where N_{FT}^i is the false track rate for the i^{th} single-sensor tracker.

5 SIMULATION-BASED PERFORMANCE EVALUATION

Our simulation is based on the simplifying assumptions that all single-sensor data rates are the same, and that the overall data flow has uniformly spaced inter-arrival times. (For example, if there are four sensors each with a 1min ping arrival interval, the single-stage tracker receives one data file each 15sec).

We set target, sensor, and tracker parameters as defined in Table 1. Our choice of Markov chain transition rates leads to a stationary probability distribution with $\pi_D^1 = 0.09$, $\pi_D^2 = 0.91$.

Table 1: Simulation parameters.

Prior velocity parameters $\sigma_{\dot{x}}$, $\sigma_{\dot{y}}$ [m/s]	1, 1
Motion uncertainty parameters q_x, q_y [m^{2}s^{-3}]	10^{-5}, 10^{-5}
Markov chain transition rates λ_{12}, λ_{21} [s^{-1}]	10^{-2}, 10^{-3}
Surveillance region $\bar{S}$ [km^2]	100
False alarm density λ [km^{-2}]	10
Number of sensors S	4
Measurement covariance parameters σ_x, σ_y [m^2]	10, 10
Ping repetition time [sec]	60
Track initiation parameters M, N	5, 6
Association gate probability	0.99
Markov chain steps K	100, 200, 400 (for convergence)

We will consider three selections for the high-detection and low-detection probabilities. First, we set these to be the same: $P_D^1 = P_D^2 = 0.5$ (case 1). Second, we have $P_D^1 = 0.3$ and $P_D^2 = 0.9$ (case 2). Third, we consider a lower low-detection probability: $P_D^1 = 0.2$ and $P_D^2 = 0.9$ (case 3). For each of these cases, we are interested in the average track confirmation time (in minutes) and the false track rate (per hour), for three tracking configurations of interest: single-sensor (S), single-stage multi-sensor (SS), and multi-stage multi-sensor (MS). Results are given in Table 2, and Figures 1-3 show the probability of track confirmation as a function of time for all tracking configurations.

Table 2: Simulation results.

Detection probabilities	Tracker configuration	Average track confirmation time [minutes]	False track rate [per hour]
$P_D^1 = P_D^2 = 0.5$	Single-sensor	27.6206	7.8141
$P_D^1 = P_D^2 = 0.5$	Single-stage	7.5594	12.5981
$P_D^1 = P_D^2 = 0.5$	Multi-stage	10.5583	31.2562
$P_D^1 = 0.3$, $P_D^2 = 0.9$	Single-sensor	37.0219	7.8141
$P_D^1 = 0.3$, $P_D^2 = 0.9$	Single-stage	26.4597	12.5981
$P_D^1 = 0.3$, $P_D^2 = 0.9$	Multi-stage	12.4071	31.2562
$P_D^1 = 0.2$, $P_D^2 = 0.9$	Single-sensor	39.1837	7.8141
$P_D^1 = 0.2$, $P_D^2 = 0.9$	Single-stage	83.9761	12.5981
$P_D^1 = 0.2$, $P_D^2 = 0.9$	Multi-stage	12.8502	31.2562

Note that, for each of the three sets of model detection settings, results are based on varying Markov chain time horizons; this is done to ensure convergence in the results. Also, note that the single-sensor performance provides insight into intermediate results, achieved after each sensor performs its own tracking and before the track-fusion stage.

In the first case, where the detection probabilities are the same, note that the single-stage track confirmation time is slightly better than the multi-stage confirmation time. This is not surprising, as there is a benefit to the high data rate seen by the single-stage tracker.

The real benefit of multi-stage tracking emerges when we consider two widely varying detection states, as in cases 2 and 3. Single-stage tracking performance is based on a detection probability that is the weighed average of the two detection probabilities, and this is a rather low value. This reflects the fact that, on average, we have difficulty in detecting target presence. On the other hand, each single-sensor tracker exploits trends in the data, since a first detection of the target indicates a high probability of being in the high-detection state, which in turn leads to a high probability of a second detection, and so forth. The track fusion stage further improves upon the single-sensor track confirmation time, by exploiting information from multiple sensors. Note that, if the low-detection probability drops low enough (as it case 3), single-sensor performance actually out-performs the single-stage multi-sensor tracker. This trend is even more dramatic as we lower the low-detection probability to 0.1 or less, which is a case of operational interest.

Of course, in addition to average track confirmation time, it is important to address the false track rate as well. Note that the results obtained do not depend on the target detection probabilities, and so are the same for all three sets of model detection settings. Also, note that while the single-stage false track rate is greater than the single-sensor rate, it is in fact smaller that what might be expected due to the increased data rate: in our case, the increase is by less than a factor of two, rather than a factor of four. This result is due to the fact that a high data rate leads to small data association gates in the tracking filter. As a result, we see that the multi-stage tracker, which combines all false tracks from the single-sensor trackers, has a larger false track rate than the single-stage tracker.

Given the multi-stage tracker's significantly better average track confirmation time, it is easy to tune the track initiation parameters M and N so that, for the same false track rate, we still achieve a significantly smaller average track confirmation time. Thus, we conclude that, faced with targets that fade in and out of favorable detection states due to source-receiver-target geometry and environmental effects, there is benefit to be gained by pursuing a two-stage tracking process. It remains to address the additional benefit that this architecture may achieve in terms of increased robustness to data registration errors.

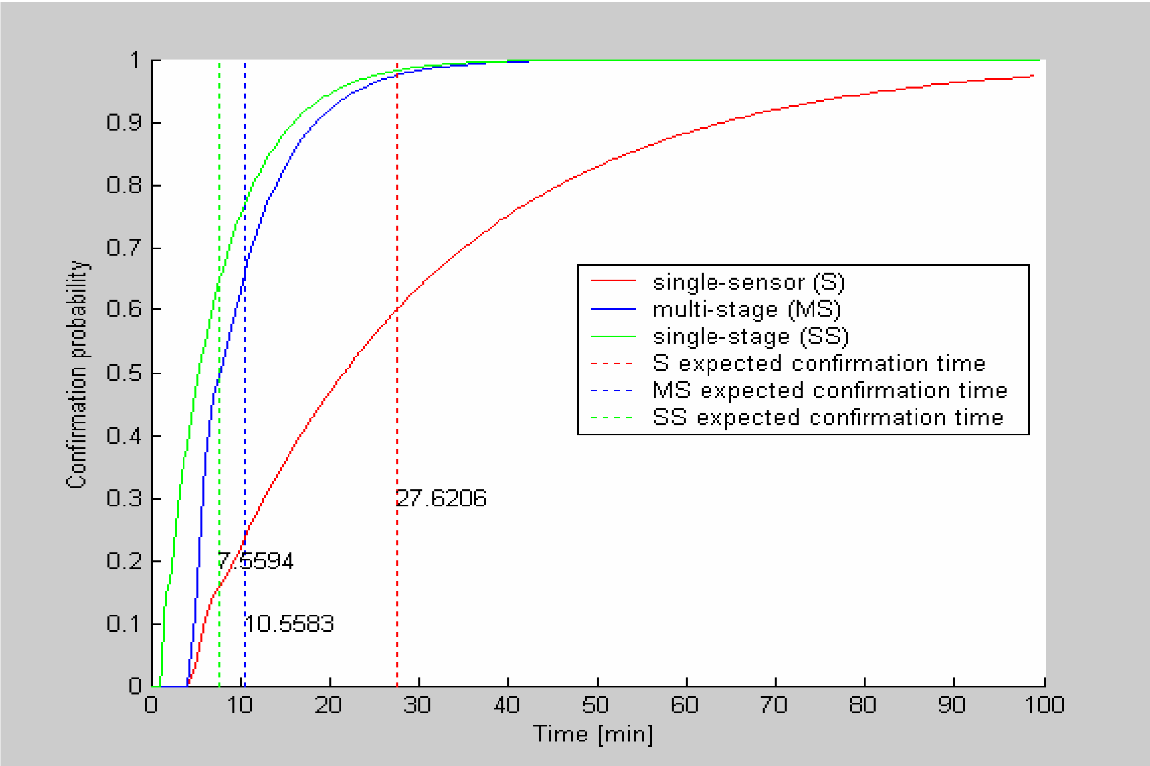

Figure 1. Probability of track confirmation over time ($P_D^1 = P_D^2 = 0.5$).

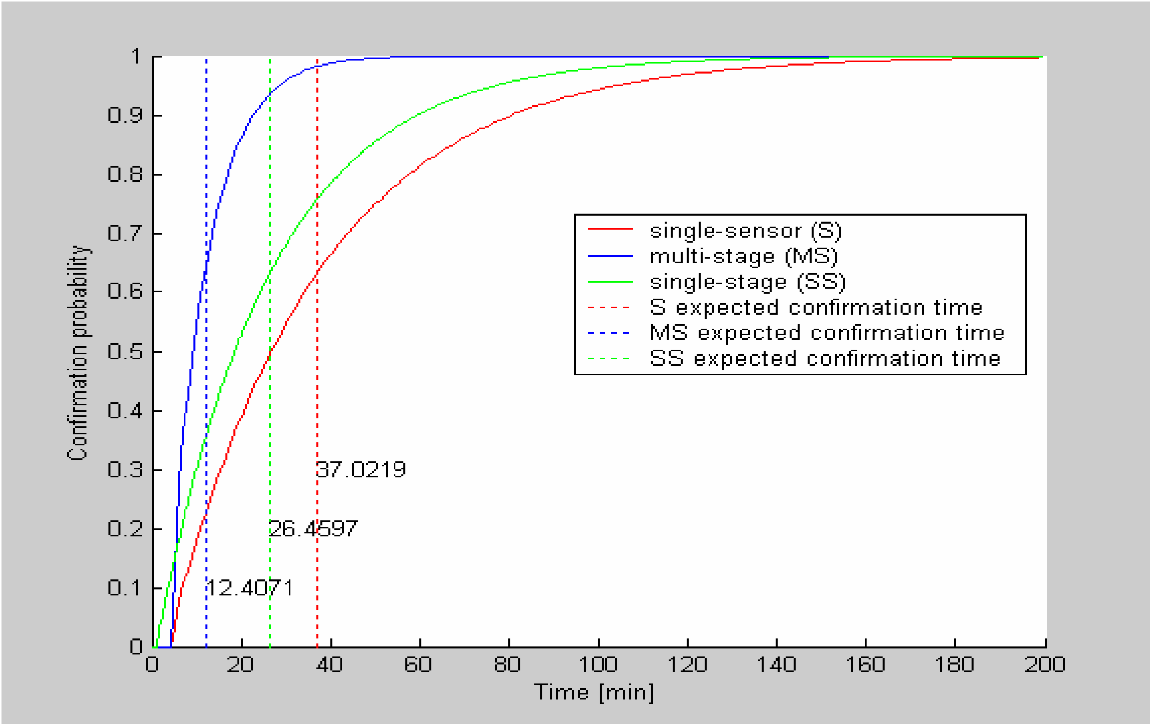

Figure 2. Probability of track confirmation over time ($P_D^1 = 0.3$, $P_D^2 = 0.9$).

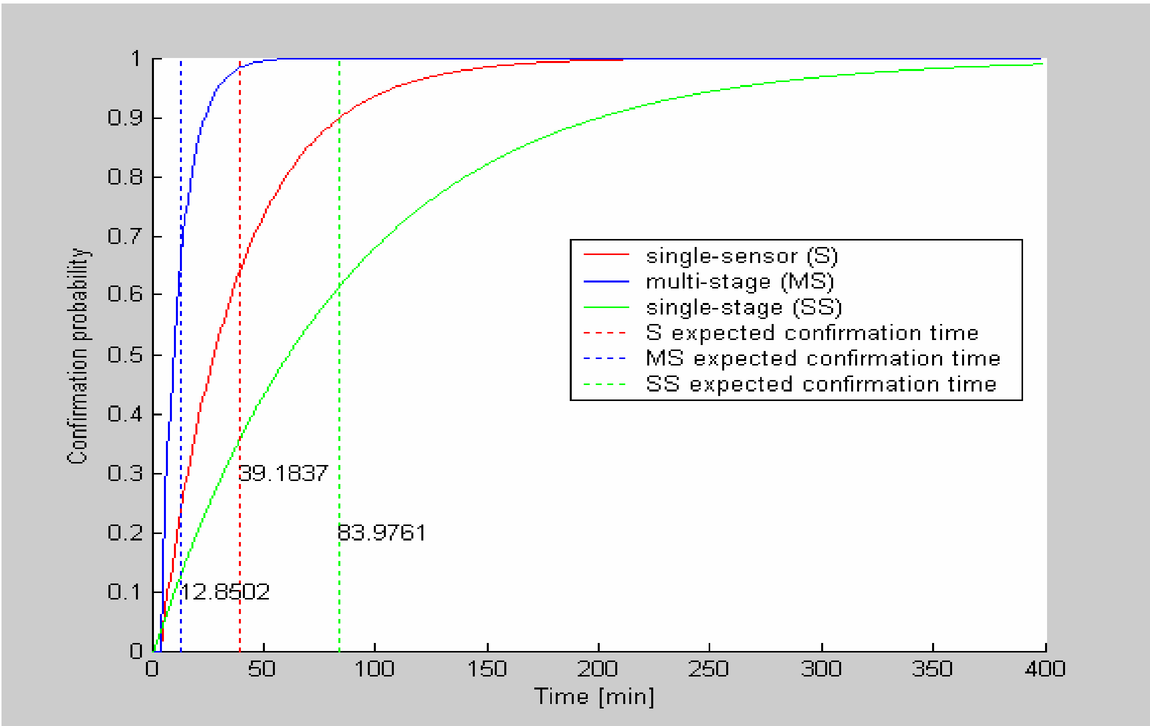

Figure 3. Probability of track confirmation over time ($P_D^1 = 0.2$, $P_D^2 = 0.9$).

6 CONCLUSIONS AND RECOMMENDATIONS

Decentralized tracking is an effective architecture for multi-sensor undersea surveillance systems with limited power and bandwidth. In addition, the multi-stage approach to sensor fusion and target tracking has the potential for improved robustness in the face of data registration errors and variable detection performance, whereby targets fade in and out of view depending on environmental conditions, source-target-receiver bistatic range, target aspect angle, etc.

This paper introduces simple models for single-stage and multi-stage trackers. These models are useful to predict tracker performance as a function of parameters that relate to target dynamics, detection performance, and tracker settings. Furthermore, these models suggest that variable detection performance is best addressed through a multi-stage tracking approach whereby single-sensor tracks are combined in a subsequent automated track fusion stage. These conclusions are based on the model simulations described in the paper. In future work, we plan to confirm these findings with actual tracker performance based on simulated as well as sea trial data.

Our tracker models, as well as our multi-hypothesis tracker described in [3], do not as yet address sensor registration errors. We plan to develop effective real-time approaches to reduce the detrimental impact of bias errors, as these may significantly reduce the effectiveness of multi-sensor fusion.

7 REFERENCES

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